

“Corrigenda in Mr. W. Shanks’s Tables ‘On the Number of Figures in the Reciprocal of a Prime.’” By Lieut.-Colonel ALLAN CUNNINGHAM, R.E., Fellow of King’s College, London. Communicated by the Secretaries.

This paper contains the result of an examination of the late Mr. W. Shanks’s MS. Tables, with above title, now deposited* with the Royal Society. These tables have been *collated*† with the following printed tables,—

1. ‘Periodische Dezimalbrüche,’ by H. Bork, Berlin, 1895. The Appendix (pp. 36–41) contains a Table (computed by Dr. F. Kessler) giving the *Residue-Index* (q), i.e., the *maximum divisor* yielding $10^{(p-1)/q} \equiv +1 \pmod{p}$ for every prime (p) $\nmid 100,000$, for which $q > 2$.

2. “Periode des Dezimalbruches für $1/p$ wo p eine Primzahl,” by H. Hertzler, printed in Grunert’s ‘Archiv der Math. und Phys.’, vol. 2, 1902, p. 249. The Table (pp. 249–251) is a *continuation* of the preceding Table for primes up to $p > 112,400$, and is arranged in the same manner.

Shanks’s MS. Tables give the *period-length* (say ξ) of $1/p$ for *all* primes from 30,000 to 120,000. The collation was effected by simply multiplying Shanks’s value of ξ by Kessler’s or Hertzler’s value of q ; the product of ξq should = $(p-1)$ in every case. The collation was, of course, only possible for such primes as have $q > 2$ (being the only ones shown by Kessler and Hertzler), thus excluding about two-thirds of the total number of primes; the ξ, q of these excluded primes are, however, easily computed when required.

By this collation a number of discrepancies (102 in all) were discovered between Shanks’s MS. and the printed German tables. The values of ξ have in all these cases been *re-computed*,‡ with the result of detecting *errata*, as follows:—

Shanks	p missing 5 ; ξ wrong 66 ; total 71	Total 102.
Kessler	p missing 8 ; q wrong 20 ; total 28	
Hertzler	p missing 1 ; q wrong 2 ; total 3	

* Part of the MS., viz., for primes from 30,000 to 60,000, is bound up with a small volume marked *Constants and Primes* (with the Press-mark 103 d 15), and the rest, viz., for primes from 60,000 to 120,000, is bound up with the ‘Royal Society Archives,’ vols. 60, 61. Mr. Shanks’s MSS. bear dates as follows:—

For primes 30,000 to 60,000, dated 1875.

For primes 60,000 to 75,000, dated 1876.

For primes 75,000 to 110,000, dated 1877.

For primes 110,000 to 120,000, dated 1880.

† By the writer of this paper, with the help of an assistant (Miss E. Cooper), by permission of the Council of the Royal Society.

‡ By the writer himself, and verified in part by Miss E. Cooper.

The table following gives the *Corrigenda* on Shanks's MS. This list is probably far from complete; there is reason to suspect a good many errors in Shanks's MS. among those primes for which $q \not\geq 2$, but, as the table has not been published, it did not seem worth while examining these.

Corrigenda on Shanks's MS. Table III [primes from 30,000 to 112,400].

Insert five primes (p) missing in MS.: 33797, 59369, 94111, 95089, 104383.

Correct the period-lengths (ξ) opposite the primes (p) as below.

p .	ξ .	p .	ξ .	p .	ξ .	p .	ξ .
33,797	8,449	65,011	21,670	86,143	1,758	103,813	17,302
34,871	1,585	70,001	35,000	86,323	14,387	104,381	104,380
42,773	289	70,867	3,937	87,121	4,840	104,383	3,866
43,753	14,584	70,921	3,546	87,151	2,075	104,707	17,451
44,893	22,446	71,821	4,788	87,517	21,879	105,367	35,122
46,153	15,384	72,559	87	87,697	29,232	105,613	26,403
46,649	686	72,661	24,220	88,003	14,667	105,929	26,482
47,093	23,546	72,871	1,735	89,689	22,422	106,031	2,305
47,711	1,835	72,901	8,100	92,107	6,579	106,921	2,430
53,857	17,952	73,351	7,335	93,319	15,553	107,647	15,378
55,021	7,860	74,413	37,206	94,111	9,411	107,837	26,959
55,681	9,280	74,687	214	95,089	23,772	109,441	9,120
55,933	27,966	78,079	3,003	95,791	15,965	110,051	22,010
57,457	2,128	79,111	7,911	96,601	4,830	110,917	18,486
57,493	14,373	80,347	13,391	96,911	4,405	110,969	27,742
58,031	4,145	82,021	16,404	98,893	24,723	111,149	148
59,369	14,842	85,411	17,082	101,051	2,150	112,249	14,031
61,007	1,034	85,447	1,818	102,793	34,264		